Simulation of coupled waves

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Abstract

A set of Scilab functions has been developed, which describes various categories of systems of coupled waves or oscillators. Co-directional and contra-directional coupling are considered. For the second case one has to introduce a function for a "partial inversion" of a matrix.

Keywords: Scilab, coupled waves, coupled oscillators

Several problems in physics imply the linear interaction of waves or of oscillators; for example the propagation in optical fibre couplers and/or Bragg reflectors, arrays of coupled resonators, acousto-optical diffraction.
The simplest case is co-directional coupling, as boundary conditions are all on one side.
Equations are of the type: $dS_i/dz=-ikS_i+\alpha(S_{i+1}-S_{i-1})$
where the amplitude S of the j-th wave, propagating in the direction z with wavevector k, interacts with the j-1-th and j+1-th wave via a coupling constant α .
The function syntax is [x]=*cowa*(n,j,alpha,z); where n is the number of interacting waves, and j is the order of the initial wave, or it is an array with the initial amplitude in each wave. If alp-ha and n are arrays of length n (number of waves) this can describe a system where each element has a different wavevector and coupling constant.
Two similar functions, *coos.sci* and *coosprop.sci* are used to calculate normal modes of an array of coupled oscillators, and the propagation of an excitation starting at one element.
For the case of acousto-optical diffraction, the equation system is called Raman-Nath equations, and a function [intensities]=*rana*(orders, distance, coupling, angle) is also included, as well as another one including the effect of reflections at the two ends.

In cases including contra-directional couplings, boundary conditions are not all at z=0, but some are at the end (z=L). .This implies the use o a function that performs a kind of partial inversion of a matrix: an equation where some unknowns are on one side of the = sign and others on the other side is transformed into an equation where all unknowns are on the same side. In the function developed here, *linear.sci*, one just have to indicate which waves have known boundary conditions on one side and which ones on the other one, and the system is solved in a way similar to the previous ones.

A function *4waves.sci* describing a system of 2 waveguides supporting 4 interacting waves as been written, and the solutions show a complex behaviour, that may have specific device applications in optical fibres. This could be easily generalised to N waveguides and 2N waves.