

A simple package for Mathieu functions

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Abstract

A package of Scilab functions has been written for the calculation of Mathieu functions. It aims at simplifying the use of these functions, which, although they have many applications, are rarely used for their complexity. This, as usual with open source software, is still evolving, with correction of bugs, extension to other less used functions, and modules for specific applications.

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Mathieu equations (ME) appear in several physical problems: a harmonic oscillator with a sinusoidally varying “elastic constant”, the inverted pendulum, alternating-gradient focussing in particle accelerators, quantum multilevel systems, eigenmodes of elliptical cavities, waveguides or membranes. They are rarely used because their definitions are rather complex, and as a consequence often not implemented in mathematical software. We aim at simplifying their use by defining a few Scilab functions that can be used as building blocks for solving problems involving Mathieu functions (MF), and can be further developed to other, less often used, functions and to specific applications.

The method used is based on diagonalisation of matrices, which suits well with a matrix-based language like Scilab. This operation is performed by a module *mathieufact*, which is used by the function *mathieufact* which calculates the various types of functions (*se*, *ce*, *Se*, *Ce*, etc), with parameters entered either directly or interactively.

For problems with elliptical boundary conditions, an auxiliary function *ell2cart* is defined, which converts elliptical coordinates to cartesian; and a function *elliptical* allows to directly find and plot the eigenmodes for given orders and ellipticity.

This will be illustrated with several examples.

In synthesis, at present the defined functions are:

```
nu=mathieufact(a,q,dim); // to calculate characteristic exponents nu (a,q constants)
[ab,d]=mathieufact(q,[dim]); // to find coefficients of expansion (dim=matrix dimension)
y=mathieufact('kind',n,z,q,[prec]); // to calculate various kind of MF ('kind'='se','ce','Se',...)
y=mathieufactS(z,a,q,ndet); // non-periodic solutions of ME
[x,y]=ell2cart(u,v,c); // conversion from elliptical u,v to cartesian x,y coordinates
[z,x,y]=elliptical(a,b,q,c); // to calculate and display elliptical modes (c=half dist. foci)
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