A New Algorithm and Its Scilab Implementation For Solution of Bordered Tridiagonal Linear Equations

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The paper puts forward a new algorithm and its Scilab implementation to solve the following bordered tridiagonal linear equations,which widely occur in engineering computation and analysis.

$$\begin{bmatrix} a_0 & b_0 & & & p_0 \\ c_1 & a_1 & b_1 & & \vdots \\ & \ddots & \ddots & \ddots & p_{n-3} \\ & & \ddots & \ddots & b_{n-2} \\ q_0 & \cdots & q_{n-3} & c_{n-1} & a_{n-1} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-2} \\ x_{n-1} \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{n-2} \\ f_{n-1} \end{bmatrix} \tag{1}$$

Let

$$\boldsymbol{p} = (p_0, p_1, \ldots, p_{n-3})^T, \boldsymbol{q} = (q_0, q_1, \ldots, q_{n-3})$$

$$\boldsymbol{u}_0 = \begin{bmatrix} \boldsymbol{p} \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{u}_1 = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$

$$\boldsymbol{v}_0 = [0, 0, \ldots, 1], \boldsymbol{v}_1 = [\boldsymbol{q}, 0, 0]$$

$$U = [\boldsymbol{u}_0, \boldsymbol{u}_1], V = [\boldsymbol{v}_0, \boldsymbol{v}_1] \tag{2}$$

and

$$A = \begin{bmatrix} a_0 & b_0 & & & \\ c_1 & a_1 & b_1 & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & b_{n-2} \\ & & & c_{n-1} & a_{n-1} \end{bmatrix}$$

then the solution of the (1)can be obtained by following steps:

Step 1. Solve $A\boldsymbol{z}_0 = \boldsymbol{u}_0$,$A\boldsymbol{z}_1 = \boldsymbol{u}_1$ to construct $Z = [\boldsymbol{z}_0, \boldsymbol{z}_1] \in R^{n\times 2}$
Step 2. Solve $A\boldsymbol{y} = \boldsymbol{f}$ to obtain $\boldsymbol{y}$
Step 3. Compute $H = [I + V^T Z]^{-1}$, which is a 2×2 matrix
Step 4. Compute $\boldsymbol{x} = \boldsymbol{y} - ZHV^T\boldsymbol{y}$ to get the final solution.

Since the three equations $A\boldsymbol{z}_0 = \boldsymbol{u}_0$,$A\boldsymbol{z}_1 = \boldsymbol{u}_1$ are independent each other, they can be solved via heterogeneous parallel computing. Hence the paper also provides a parallel solution for the equations.

Detail mathematical deductions of the algorithm are presented and a Scilab implementation of the algorithm are also introduced with experiments.

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