The Research for Parameters Estimation of Distractor Model

Anyu Zhang[1,2] Xiaoyao Xie* [2,1]*Member IEEE*

1. *School of Computer Science and Technology, Guizhou University*
2. *Key Laboratory of Information and Computing Science of Guizhou Province, Guizhou Normal University*

zhanganyu2004@126.com,xyx@gznu.edu.cn (corresponding author: Xiaoyao Xie)

Abstract

Item Response Theory (IRT) is also sometimes called latent trait theory. This is a modern test theory (as opposed to classical test theory). It is not the only modern test theory, but it is the most popular one and is currently an area of active research [4]. Multiple choice items consist of a stem and a set of options. The stem is the beginning part of the item that presents the item as a problem to be solved. The options are the possible answers that the examinee can choose from, with the correct answer called the key and the incorrect answers called distractors [3]. Currently, multiple-choice models are base on numerous response models. Distractor model a new response model for multiple-choice, which is similar to the development of the numerous response model (NRM), and consists with distractor rejection model (DLT) [7] (2008).

How to estimate the ability parameter of testee is a problem. Firstly, we suppose we get parameters of response item, we can use condition maximum likelihood estimation.

Suppose that the parameters of options of the j *th item is known, include* a_{jk}, b_{jk}, c_{jk}, *the number of items is* J, *the number of testees is* I, *the response matrix is* $U=(u_{ij})$, *and* $\theta=(\theta_1,\theta_2,\cdots,\theta_I)$, *where* $u_i=(u_{i1},u_{i2},\cdots,u_{iJ})$ *is the response model of the* i *th testee[8],* $u_{ij} = P_{ij}(x_j = k'|\theta)$, *and* θ_i *is the ability of* i *th testee.* $P_{ij}(x_j=k'|\theta)$ *is the probability that the* i *th testee have selected* k' *th option in* j *th item. Then the likelihood function is*

$$L(\theta;U)=\prod_{i=1}^{I}\prod_{j=1}^{J}P_{ij}(x_j=k'|\theta) \quad (1)$$

For getting extremum of θ_i, *show the first derivative of (8). Then*

$$\frac{\partial \ln L(\theta_i;U)}{\partial\theta_i}=\sum_{j=1}^{J}(P_{ij}(x_j=k'|\theta_i))^{-1}T_j(\theta_i) \quad (2)$$

Where $$T_j(\theta_i)=\frac{Da_{jk}(1-c_{jk})E_{jk}}{(1+c_{jk}E_{jk})^2\sum_{k'=1}^{m}w_{jk'}}+\frac{w_{jk}}{(\sum_{k'=1}^{m}w_{jk'})^2}\sum_{k'=1}^{m}\frac{Da_{jk'}(1-c_{jk'})E_{jk'}}{(1+c_{jk'}E_{jk'})^2} \quad (3)$$

Let the first derivative of (2) equal to 0, can get the log likelihood equation (3)

$$\sum_{j=1}^{J}(P_j(x_j=k'|\theta_i))^{-1}T_j(\theta_i)=0 \quad (3)$$

The solution of equation (3) is the maximum likelihood estimation value of θ_i.

Given parameters of options, we can estimate the ability of testee. The model and estimation show that distractor model does not base on items but options, and then we can get more information.

*In fact, we usually can't get parameters of options. So a problem is how to get parameters of options? Using joint maximum likelihood estimation (JMLE) [**9**], we can estimate parameters of option. Next estimate ability of testees. Repeat it until the difference between twice of recurrence is less than prior error.*

Now, solution equation (2) is needed. At same times, it is necessary to resolve equals (2). Equations (2) usually use Newton method. Suppose

$$F(X)=\left(\frac{\partial(\ln L(U))}{\partial a_{jk}},\frac{\partial(\ln L(U))}{\partial b_{jk}},\frac{\partial(\ln L(U))}{\partial c_{jk}}\right) \tag{4}$$

(2) is an odd non-linear equations.

Mark item parameters vector as X *, first-order partial derivative matrix* $F(X^{(t)})$*, second-order partial derivative matrix* $DF(X^{(t)})$ *, equations can be written as*

$$X^{(t+1)}=X^{(t)}-[DF(X^{(t)})]^{-1}F(X^{(t)}) \tag{5}$$

Where $[DF(X^{(t)})]^{-1}$ *is the inverse matrix of* t *th step calculation.*

From the formula and (5), it is hard to calculate $[DF(X^{(t)})]^{-1}$*, so it is necessary to find a new method to solve equation. For finding a new method, we can define a formula[2].*

$$X^{(t+1)}=X^{(t)}-(A^{(t)})^{-1}F(X^{(t)}) \tag{6}$$

By Quasi-Newton method, construct iterative function as

$$\begin{cases} X^{(t+1)}=X^{(t)}-(A^{(t)})^{-1}F(X^{(t)}) \\ p^{(t)}=-(A^{(t)})^{-1}F(X^{(t)}) \\ q^{(t)}=F(X^{(t+1)})-F(X^{(t)}) \\ A^{(t+1)}=A^{(t)}+\dfrac{(q^{(t)}-A^{(t)}p^{(t)})(p^{(t)})^T}{\| p^{(t)}\|_2^2} \end{cases} \tag{7}$$

where $\| p^{(t)}\|_2$ *is the vector norm of* $p^{(t)}$*.*

Item response models used for educational and psychological measures vary considerably in their complexity, especially multiple-choice. In this paper, we proposed the estimation method of distractor model. The maximum likelihood function and its joint maximum likelihood extraction are provided. Based in the current analysis, we can research initial values and the arithmetic for distractor models. We also can combine with MMLE/EM and Bayes estimation to analyze the new results.

Key words: Multiple-choice, Parameters estimate, Distractor model

Reference

[1] Baker, F.B. Equating tests under the graded response models. *Applied Psychological Measurement*, 1992,16, 87-96

[2] Guan Zhi, Lu Jingpu. Numerical Analysis Basis. Higher Education Press. 1998

[3] http://en.wikipedia.org/wiki/Multiple_choice

[4] http://luna.cas.usf.edu/~mbrannic/files/pmet/irt.htm

[5] Jee-Seon Kim. Using the distractor categories of multiple-choice items to improve IRT linking. Paper presented at the 2006 NCME annual meeting, San Francisco, CA

[6] Kehoe, Jerard. Writing multiple-choice test items *Practical Assessment, Research & Evaluation, 4*(9). February 12, 2008.

[7] Randall D.Penfield&Jimmy de la Torre, A new response model for multiple-choice items. Paper presented at the annual meeting of the National Council on Measurement in Education,New York City, 2008

[8] Qi Shiqing. The application of modern measurement theory in the examination. Central China Normal University Pressed. 2003(8)

[9] Wei Zongshu. Probability Theory and Mathematical Statistics. Higher Education Press. 1983